

An applied stochastic model of the quality–quantity trade-off in the public health care sector

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Abstract It is a striking feature of the many of the developing country public service sectors that the sectors in question often overproduce the quantity of services but underproduce the quality. This feature, which is exemplified in this paper, is rooted in a wide spectrum of economic and sociological factors ranging from the economic and sociological profile of the service receiving people to the asymmetric density of service-receiving population across their regions. This feature, we conjecture, is a source of a considerable degree of suboptimality in some of the developing countries. If our conjecture is correct, correcting such suboptimalities is likely to yield significant welfare improvements that could help speed up the process of development in the underdeveloped regions of the world. To analyze the suboptimalities in question, we will first develop a concept (and a model) of *optimal quality in the public service sector*, which indicates the level of quality that maximizes expected public satisfaction subject to available resources. Resources are used in an efficient manner to produce the service in question. The concept and the model in the paper make a needed contribution to the quality discourse by presenting a way of determining the quality improvements (or adjustments) necessary to achieve optimum in the public service sector. The paper presents an application (a case study) of this new concept in the public healthcare sector in Turkey, and explores the differences between the actual and optimal quality in the sector in question. It turns out that there is a considerable difference between the actual and optimal levels of quality (as well as those of quantity) in the Turkish public healthcare sector in an overpopulated city (Istanbul), indicating a significant *overproduction of quantity* and *underproduction of quality*. Thus, to achieve the optimal levels, the sector should increase quality and reduce quantity by a considerable margin. The quantified differences (gaps) between actual and optimal levels point out a considerable room for welfare improvement. Optimum-seeking adjustments closing these gaps could be shown to lead to considerable satisfaction and welfare gains, the measurement of which is worthy of future research.

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1 Introduction

The literature on the public service sector includes a rich spectrum of works ranging from the static explanations of the public sector problems to dynamic explorations of the intertemporal trajectory of these problems. Among these works are Shi (1994), Corry (1997), Domberger and Jensen (1997), Duncombe et al. (1997), Haskel et al. (1998), World Bank (1998), Bartel and Harrison (1999), Downes and Figlio (1999), Karlaftis and MacCarthy (1999), Tanzi (2000), Kara et al. (2002a,b, 2003), and West (2004).

In the literature, there is fundamental criticism of many of the public service sectors in the developing and developed countries that the sectors in question often produce low quality- service. Dynamic examinations of some of these sectors (such as Kara et al. 2003) have shown that these sectors are often trapped into a low performance-low quality equilibria. Although the works in question point out possible ways out of such equilibria, the central question still remains unanswered: what is the desirable level of quality for the services produced by the public sector? Since quality in the public service sector is not an end in itself, rather it is a means to achieving higher public satisfaction, the main task should be to find the level of quality (as well as that of quantity) that would maximize public satisfaction under the society's resource constraints. The constrained optimum is likely to be susceptible to trade-offs between quality and quantity which we will explore in this paper.

In the second section of the paper, we will develop a concept and model of optimal quality (as well as quantity) in the public service sector. The third section will present an empirical application of the model. The concluding remarks will follow in the fourth section.

2 Optimal quality in the public service sector

There are many different definitions of the concept of quality. Some definitions have focused on the issue of the “compliance with policies and procedures or conformance to certain standards of excellence,” some on the “fitness for use or meeting and exceeding customers' expectations,” and some on other features.¹ However it is defined, the concept of quality has gained a primary importance in the modern business theory, and a considerable amount of intellectual effort has been invested in its measurement. Various advances made on the issue of measurement could be followed from the literature.²

¹ For a summary of the alternative conceptions of quality, see Evans and Dean (2003) and Rao et al. (1996).

² Among the works on quality are Crosby (1980), Deming (1981–1982), Parasuraman et al. (1985, 1988, 1991), Juran (1986), Carman (1990), Babakus and Boller (1992), Cronin and Taylor (1992, 1994), Juran and Gyra (1993), Teas (1994), Taylor and Cronin (1994), Fayek et al. (1996), Rao et al. (1996), Kanji and Yui (1997), Yavas et al. (1997), Bloemer et al. (1999), Kanji et al. (1999), Dabholkar et al. (2000), Caruana et al. (2000), Kara (2000), Kara et al. (2002a, 2002b, 2003), Kara et al. (2005), Li and Collier (2000), Lim and Tang (2000), Sivadas and Baker-Prewitt (2000), Yavas and Shemwell (2001), Andaleeb (2001), Brady and Cronin (2001), Kanji and Sa (2003), Kaya (2000), Savas et al. (2002), and Scrivens (1995).

As important as the issue of quality measurement might be to the business organizations and customers, the question of what level of quality is best for such organizations and customers remains to be answered. To answer this question for *public service-producing organizations*, we will propose a new concept of optimal quality for the public service sector, which we will define as follows:³

*Optimal quality for a public service is the level of quality that maximizes (expected) public satisfaction subject to available resources. Resources are used in an efficient manner to produce the service in question.*⁴

To determine the optimal quality in question, we will develop an easily accessible model, the basic features of which are as follows:

2.1 Public (Customer) satisfaction (utility)

Public satisfaction could be represented in a number of ways. For the sake of simplicity and analytical convenience, we will represent public satisfaction in terms of the utility (satisfaction) of a representative member of the public (customer) receiving the service. Alternatively a “community utility function” or a “social welfare function” could serve the same purpose as well. To theorize about customers’ satisfaction, we will make use of the utility theory developed and widely used in economics. The concept of utility, which is defined as the satisfaction derived from consuming or possessing goods or services, and which is represented by a real-valued utility function mapping the goods/services consumed to the satisfaction derived, has served, in economics, as a fundamental theoretical construct in formulating consumers/customers’ wants or preferences. We will follow the economics profession’s utility-theoretic, analytical conventions about consumers/customers’ wants/preferences, but formulate a peculiar kind of utility function where “not only the quantity of service but also the quality of service is a direct item of choice” (for an example of such a utility function, see [Nicholson 1998](#), p. 96). The rationale for employing such a utility function resides in the very objective of this paper, namely the objective of determining the level of quality (as well as quantity) of the service that would maximize the customers’ satisfaction.

Suppose that a *representative customer’s* satisfaction (utility) (U), derived from services (x_1, \dots, x_m), is a function of the quantities, $q_{1i}, i = 1, \dots, m$, and the associated qualities, $q_{2i}, i = 1, \dots, m$, of the services consumed,

$$\text{i.e., } U = U(q_{11}, \dots, q_{1m}, \dots, q_{21}, \dots, q_{2m}), \quad \text{where } \frac{\partial U}{\partial q_{1i}} > 0, \\ \frac{\partial U}{\partial q_{2i}} > 0, \quad q_{ji} \in R, \quad j = 1, 2; \quad i = 1, \dots, m^5,$$

where R is the set of real numbers. Hence, we have a framework with multiple quan-

³ Juran has made use of some notion of optimality in his concept of quality, but his notion is confined to a limited producer’s optimization in the form of cost minimization, excluding customer’s optimization.

⁴ We have also developed a concept of optimal quality for private sector. See [Kara \(2005\)](#). Note that the concept of quality here represents “service quality.” Alternatively, however, it could be taken to represent “total quality in organizations.”

⁵ Although, we assume positive marginal utilities throughout the domain of the utility function, allowing zero or negative marginal utilities in some subsets of the domain is not analytically intractable.

tities and multiple qualities.⁶ Let $(q_{11}, \dots, q_{1m}, q_{21}, \dots, q_{2m})$ be a $2m$ -variate normal random variable with a joint probability density function, $f(q_{11}, \dots, q_{1m}, q_{21}, \dots, q_{2m})$. Expected utility is given by

$$E[U(q_{11}, \dots, q_{2m})] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} U(q_{11}, \dots, q_{2m}) f(q_{11}, \dots, q_{2m}) dq_{11}, \dots, dq_{2m}.$$

For analytical convenience and simplicity, however, we focus, in this paper, on the case with one service with varying quantity (q_1) and quality (q_2).⁷ Here q_1 and q_2 could also be conceived to be aggregated proxies for multiple disaggregated quantity and quality dimensions. Thus, the expected utility function takes the form:

$$E[U(q_1, q_2)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(q_1, q_2) f(q_1, q_2) dq_1 dq_2, \quad \text{where } \frac{\partial U}{\partial q_1} > 0, \quad \frac{\partial U}{\partial q_2} > 0.$$

2.2 Quality-quantity frontier

Suppose that public firms (institutions) have certain amounts of resources at their disposal to produce varying quantity levels of the service at varying levels of quality. Since the amounts of resources available to the public institutions is set by the local or central governments, it is reasonable to assume that they are exogenously fixed. Using the information about resources and service technology, we can derive, what we will call the *quality–quantity frontier*, which represents the combinations of various levels of quality and quantity that firms could, given technology, produce using the relevant available resources to the fullest extent possible. Let $t(q_{11}, \dots, q_{1m}, q_{21}, \dots, q_{2m}) = 0$ represent, in implicit form, the *quality–quantity frontier*. The overall optimization problem is to maximize expected customer’s satisfaction subject to the this frontier,

$$\begin{aligned} \text{i.e., Maximize } & E[U(q_{11}, \dots, q_{1m}, q_{21}, \dots, q_{2m})], \\ \text{subject to } & t(q_{11}, \dots, q_{1m}, q_{21}, \dots, q_{2m}) = 0. \end{aligned}$$

For simplicity, we will concentrate on the case with one service with one quantity–one quality dimensions, in which case the quality–quantity frontier would take the form of $t(q_1, q_2) = 0$. The derivation of this frontier is illustrated in Appendix 1. The optimization problem in this reduced case will be:

$$\begin{aligned} \text{Maximize } & E[U(q_1, q_2)], \\ \text{subject to } & t(q_1, q_2) = 0. \end{aligned}$$

The first-order conditions of optimization yield:⁸

$$\frac{\frac{\partial E(U)}{\partial q_1}}{\frac{\partial E(U)}{\partial q_2}} = \frac{\frac{\partial t}{\partial q_1}}{\frac{\partial t}{\partial q_2}}.$$

⁶ The analysis could easily be extended so as to take into account the cases where a service may have multiple quality attributes.

⁷ The difference between a framework with one service and the one with multiple services is only a matter of analytical calculations. Extending the one service-framework to the case of multiple services with multiple quantity–multiple quality dimensions is tedious but not difficult to undertake.

⁸ The second-order conditions are satisfied if the determinant of the bordered Hessian matrix associated with this optimization problem is positive. For an exercise, see Kara (2000).

The right-hand side of this equality is nothing but the marginal rate of transformation between quality and quantity, and as such—it can be shown that—it is equal to the marginal cost ratios.⁹ Thus, in order to maximize public satisfaction subject to the quality-and-quantity-possibilities that the society’s resource constraints and technology permit, we should find the levels of quality and quantity (namely, optimal quality, and quantity), which are on the quality–quantity frontier and, for which the equality of expected marginal utility and marginal cost ratios is met.

In order to find explicit expressions for optimal quality and quantity, let customer’s satisfaction (utility), probability density function of quality and quantity, and costs be of the following forms:¹⁰

$$\text{Customer satisfaction: } U = -\exp\{-\alpha_1 q_1 - \alpha_2 q_2\}, \quad \alpha_1 > 0, \quad \alpha_2 > 0.$$

$$\text{Probability density function: } f(q_1, q_2) = \frac{1}{2\pi\sigma_1\sigma_2} \exp\left\{-\frac{1}{2}\left[\left(\frac{q_1-\mu_1}{\sigma_1}\right)^2 + \left(\frac{q_2-\mu_2}{\sigma_2}\right)^2\right]\right\},$$

where μ_1, μ_2, σ_1^2 , and σ_2^2 are means and variances of q_1 and q_2 , respectively.

$$\begin{aligned} \text{Costs: } C(q_1) &= B_1 q_1^2, \\ C(q_2) &= B_2 q_2^2. \end{aligned}$$

Based on these functional forms, we derive the following expressions for optimal mean quantity and optimal mean quality (see Appendix 2):

$$\begin{aligned} \text{Optimal mean quantity: } \mu_1^* &= \left(\frac{c}{\beta_1 + \left(\frac{\alpha_2\beta_1}{\alpha_1\beta_2}\right)^2 \beta_2}\right)^{1/2}, \\ \text{Optimal mean quality: } \mu_2^* &= \left(\frac{c}{\beta_2 + \left(\frac{\alpha_1\beta_2}{\alpha_2\beta_1}\right)^2 \beta_1}\right)^{1/2}, \end{aligned}$$

where c is the integration constant defined in Appendix 2.

A close examinations of these expressions reveals a theoretically expected, fundamental property of optimal mean quality and quantity, namely that determinants of optimal mean quality and quantity are not only cost (or supply) based; rather there are both demand-based and supply based factors affecting (determining) optimal mean quality and quantity. The demand-based factors include the satisfaction (utility)-based parameters, α_1 and α_2 , which reflect the weights of quantity and quality in the customer’s satisfaction. The supply based factors include the cost-based parameters, B_1 and B_2 , which reflect the weights of quantity and quality in the overall cost, and a resource-based parameter, c , the value of which is based on the amount of available

⁹ A more proper wording of the overall efficiency condition would be as follows: the expected marginal utility ratio (i.e., the expected marginal rate of substitution of q_1 for q_2) should be equal to the marginal rate of product transformation of q_1 for q_2 along the quality–quantity frontier, which is also equal to the marginal cost ratios.

¹⁰ The reason for choosing the designated utility function is as follows: the utility function in question, which is among the widely used utility function forms in economics, fits well into the set of data we have used. Needless to say, the particular functional form chosen affects the empirical results. As shown in the utility-relevant regression results in Sect. 3, R^2 is quite high, and the coefficients have the theoretically expected signs. Hence, we can reasonably argue that the “goodness” of the empirical results vindicate the appropriateness of the particular utility function form we have chosen.

resources.¹¹ From the expressions above, we can easily infer the direction of the effects of demand-based and supply based factors on optimal mean quality and quantity: the higher is the weight of quality (relative to that of quantity) in customer's satisfaction, the higher is the optimal mean quality. The higher is the weight of quality in total costs, the lower is the optimal mean quality. The higher is the amount of resources, the higher is the optimal mean quality. A similar analysis holds also for optimal mean quantity.

3 An application: empirical results

We collected data on such basic variables as quality, quantity, customer satisfaction, and costs associated with the health care services in a subset including 13 cases of the public hospitals in Istanbul, Turkey. Variables are measured on a five-point scale with 1 being "very low" and 5 being "very high." The measurement of easily observable variables such as quantity and costs is done in a fairly precise manner with a reasonably small margin of error. However, the measurement of variables such as quality and satisfaction, which are not so easily observable, is based, in part, on the subjective assessments of the individuals involved, and as such it may include a relatively wider margin of error. For instance, to measure the quality of the service and the service satisfaction, people at the institutions are asked to rate the quality and perceived satisfaction on a scale of 1–5. Presuming that individuals would not often make systematic assessment errors, the subjective assessments could be taken to be reasonably good proxies for the values of imprecisely observable variables.

Based on the data available, we estimated the following regression equations for customer satisfaction and costs:¹²

Customer satisfaction: using a linear approximation of the utility function, we will employ the following equation:¹³

$$U^* = \alpha_1 q_1 + \alpha_2 q_2 + u,$$

where $U^* = U + 1$

$$\text{Costs: } C(q_1) = B_1 q_1^2 + v,$$

$$C(q_2) = B_2 q_2^2 + z,$$

where u , v , and z are the disturbance terms. The total costs are $C(q_1, q_2) = C(q_1) + C(q_2)$. The regression results are as follows:

$$U^* = 0.229 q_1 + 1.068 q_2, \quad R^2 = 0.95,$$

(1.483) (4.917)

$$C(q_1) = 0.629 q_1^2, \quad R^2 = 0.89,$$

(9,401)

¹¹ The Juran's notion of *optimal conformance*, which is somewhat close to, but narrower than, the notion of *optimal quality* defined here, is only cost-based. It does not take into account the demand-based (satisfaction-based) factors. Nor does it take into consideration the resource-based constraints.

¹² In the measurement of the cost of quality, we also made use of the data on the ratio of quality costs to total revenue.

¹³ For a demonstration of the process of linearization for the utility function in question, see Kara (2000).

$$C(q_2) = 0.899 q_2^2, \quad R^2 = 0.57, \quad (2.824)$$

where t -statistics are given in parentheses.

The levels of optimal mean quantity and quality, μ_1^* and μ_2^* are derived in Appendix 2. These are:

$$\mu_1^* \cong 1.37$$

and

$$\mu_2^* \cong 4.49.$$

A few points on these results are in order:

1. $\alpha_1 = 0.229 < \alpha_2 = 1.068$, i.e., in customer's satisfaction, the weight of quality is greater than that of quantity, i.e., customers place a higher value on quality than quantity.
2. $\beta_1 = 0.629 < \beta_2 = 0.899$, i.e., in total costs, the weight of quality is greater than that of quantity. Thus, increasing quality is costlier than increasing quantity.
3. Dividing the expressions for optimal quality and quantity, we get,

$$\frac{\mu_2^*}{\mu_1^*} = \left(\frac{\alpha_2 \beta_1}{\alpha_1 \beta_2} \right).$$

Thus, the scale-value of optimal mean quality relative to that of optimal mean quantity would depend on the relative weights of quality and quantity in customer's satisfaction and the relative weights of quality and quantity in total costs. Since in our empirical case, the effects of relative satisfaction induced by quality compared to quantity outweigh the effects of relative cost of quality compared to quantity, the scale-value of optimal quality turns out to be higher than that of optimal quantity.

4. For policy analysis, the most important information would be the differences between optimal values of mean quality and quantity derived above and their actual values. For the sample of the hospitals under consideration, the actual values of quantity and quality averages are as follows:

Actual average quantity: $q_1 = 4$,

Actual average quality: $q_2 = 2.833$.

A comparison of the actual mean values with the optimal mean values derived above demonstrates an expectedly striking feature of the public Turkish healthcare sector in Istanbul that we examined, namely that there is a considerable difference between the actual and optimal levels of quality as well as those of quantity. More explicitly, the actual mean quantity is considerably higher than the optimal mean quantity, but the actual mean quality is considerably lower than the optimal mean quality, indicating a significant *overproduction of quantity* and *underproduction of quality*. Thus, to achieve the optimal levels, the sector should increase quality and reduce quantity by a considerable margin—margins reflected by the numbers above. The quantified differences (gaps) between actual and optimal levels point out a considerable room for welfare improvement. Optimum-seeking adjustments closing these gaps could lead to considerable satisfaction and welfare gains, the measurement of which is worthy of future research.

4 Concluding remarks

For the specific subset of the public sector under examination, the differences between optimal and actual values in the form of an overproduction of quantity and underproduction of quality have turned out to be quite significant. This reflects a high degree of suboptimality and imperfection in the sector in question. The welfare losses associated with such a suboptimality are significant as well. The notion of concretely determinable optimum worked out in this paper enables us to determine the level of quantity and quality adjustments necessary to achieve the optimum, and point out the welfare gains such optimum-seeking adjustments could bring about. Thus the concept and the model developed in the paper would prove to be valuable to the public enterprise (or institution) managers and policy makers in their efforts to increase customers' (or public) satisfaction in particular and society's welfare in general.

Appendix¹⁴

Appendix 1: The quality–quantity frontier for one service with one quality–one quantity dimensions.

What we call the *quality–quantity frontier* indicates combinations of various levels of quality and quantity that firms could, given technology, produce using the relevant available resources to the fullest extent possible. In other words, it shows, for each level of the quantity of the service, the maximum quality that can be produced (obtained). Let $a_j, j = 1, \dots, n$, be the amounts of n inputs (resources) that can be used to produce varying quantities of the service at varying qualities. Let $y_{1j}, j = 1, \dots, n$ be the amounts of inputs used in the production of various quantities of the service (q_1), and $y_{2j}, j = 1, \dots, n$, be the amounts of inputs employed in securing (producing) various levels of qualities (q_2). Supposing that the relevant resources are fully used in producing a certain quantity of the service or in securing a certain quality, we have the following resource constraints:

$$\sum_{i=1}^2 y_{ij} = a_j, \quad j = 1, \dots, n.$$

We will introduce what we will call the quantity and quality production functions, f^1 and f^2 , which represent, given technology, the maximum quantity and quality of the service that can be produced from each bundle of inputs, i.e.,

$$q_1 = f^1(y_{11}, \dots, y_{1n})$$

and

$$q_2 = f^2(y_{21}, \dots, y_{2n}).$$

In order to find, with full utilization of resources, the maximum quality of the service for each level of the quantity, we need to solve the following optimization problem:

¹⁴ Some of the derivations follow Kara (2005).

$$\begin{aligned} &\text{Maximize } q_2 = f^2(y_{21}, \dots, y_{2n}) \\ &\text{subject to } f^1(y_{11}, \dots, y_{1n}) = q_1 \end{aligned}$$

and

$$\sum_{i=1}^2 y_{ij} = a_j, \quad j = 1, \dots, n.$$

The solution to this optimization problem yields the *quality–quantity frontier* in either implicit form (i.e., $t(q_1, q_2) = 0$) or in explicit form (i.e., $q_2 = g(q_1)$).

Using some properties of *quality–quantity frontier*, we can derive the frontier in an alternative manner: it is straightforward to observe that the slope of the frontier is nothing but the negative of the marginal rate of transformation of q_1 for q_2 , which is equal to the ratio of marginal costs. That is to say,

$$\frac{dq_2}{dq_1} = - \frac{\frac{\partial C}{\partial q_1}}{\frac{\partial C}{\partial q_2}}.$$

This equation, through integration and with some properly specified initial conditions, yields the *quality–quantity frontier*. An example of such a derivation in the context of the empirical case under consideration will be given in Appendix 2.

In the case of multiple services with multiple quantity–multiple quality dimensions, it is easy to show that the partial derivative of one variable with respect to another is equal to the negative of the ratio of associated marginal costs, which yield a system of partial differential equations, the solution of which, with some properly specified initial conditions, results in the *quality–quantity frontier in the multiple-dimension-case*.

Appendix 2 Calculation of the parametric expressions and empirical values of the optimal quality and quantity

Optimal levels of quality and quantity are those that maximize utility subject to the *quality–quantity frontier*. Based on the theory and the empirical results of the paper, we will first derive the *quality–quantity frontier* and then proceed to find the levels of q_1 and q_2 that maximize utility subject to that frontier.

2.1 Derivation of the *quality–quantity frontier*

Using the model and empirical results, we can now explicitly state the slope of the frontier, which is:

$$\begin{aligned} \frac{dq_2}{dq_1} &= - \frac{\frac{\partial C}{\partial q_1}}{\frac{\partial C}{\partial q_2}} = - \frac{2 \times \beta_1 q_1}{2 \times \beta_2 q_2} \\ &\Rightarrow 2 \times \beta_2 q_2 dq_2 = -2 \times \beta_1 q_1 dq_1. \end{aligned}$$

Integrating both sides and rearranging the terms, we get,

$$\beta_1 q_1^2 + \beta_2 q_2^2 = c,$$

where c is the integration constant. Substituting the empirical values, $\beta_1 = 0.629$, and $\beta_2 = 0.899$, we obtain,

$$0.629q_1^2 + 0.899q_2^2 = c.$$

To find an approximate value for c , we will stipulate an initial condition making use of the minimal quality requirement for health care services. It turns out that the perceived minimally acceptable level of quality in the hospitals in question could, on average, be represented by an approximate scale of 2. At a average quality scale of 2, the maximum quantity of the service, represented by a scale of 5, could, on average, be provided,

$$\text{i.e. for } q_2(= \mu_2) \cong 2, \quad q_1(= \mu_1) \cong 5.$$

For these values of q_1 and q_2 , $c = 19.321$.¹⁵ Thus the mean quality–quantity values compatible with the quality–quantity frontier requires that:

$$0.629\mu_1^2 + 0.899\mu_2^2 = 19.321.$$

2.2 Maximization of utility subject to the *quality–quantity frontier*

$$\begin{aligned} \text{Maximize } E[U(q_1, q_2)] &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(q_1, q_2)f(q_1, q_2), dq_1 dq_2 \\ \text{subject to: } &0.629\mu_1^2 + 0.899\mu_2^2 = 19.321, \end{aligned}$$

where

$$\begin{aligned} U(q_1, q_2) &= -\exp\{-\alpha_1q_1 - \alpha_2q_2\} \\ f(q_1, q_2) &= \frac{1}{2\pi\sigma_1\sigma_2} \exp\left\{-\frac{1}{2}\left[\left(\frac{q_1 - \mu_1}{\sigma_1}\right)^2 + \left(\frac{q_2 - \mu_2}{\sigma_2}\right)^2\right]\right\}. \end{aligned}$$

Through integration and rearrangement of terms and through the use of properties of probability density functions, we get the following expression for the expected utility:

$$E[U(q_1, q_2)] = -\exp\{-\alpha_1(\mu_1 - 1/2\sigma_1^2) - \alpha_2(\mu_2 - 1/2\sigma_2^2)\}.$$

The langrangian expression associated with this optimization problem is:

$$L = -\exp\{-\alpha_1(\mu_1 - 1/2\alpha_1\sigma_1^2) - \alpha_2(\mu_2 - 1/2\alpha_2\sigma_2^2)\} + \lambda(c - \beta_1\mu_1^2 - \beta_2\mu_2^2).$$

The first-order conditions are:

$$\begin{aligned} \frac{\partial L}{\partial \mu_1} &= \alpha_1 \exp\left\{-\alpha_1(\mu_1 - 1/2\alpha_1\sigma_1^2) - \alpha_2(\mu_2 - 1/2\alpha_2\sigma_2^2)\right\} - \lambda \times 2 \times \beta_1\mu_1 = 0, \\ \frac{\partial L}{\partial \mu_2} &= \alpha_2 \exp\left\{-\alpha_1(\mu_1 - 1/2\alpha_1\sigma_1^2) - \alpha_2(\mu_2 - 1/2\alpha_2\sigma_2^2)\right\} - \lambda \times 2 \times \beta_2\mu_2 = 0, \\ \frac{\partial L}{\partial \lambda} &= c - \beta_1\mu_1^2 - \beta_2\mu_2^2 = 0. \end{aligned}$$

¹⁵ Note that, for a greater amount of resources, maximum quantity would be obtained in combination with a level of quality higher than the minimally accepted level, in which case c would attain a higher value. That is to say, the value of c is based on the amount of available resources.

Solving this set of equations, we get the following parametric expressions for optimal mean quantity and optimal quality:

$$\text{Optimal mean quantity: } \mu_1^* = \left(\frac{c}{\beta_1 + \left(\frac{\alpha_2 \beta_1}{\alpha_1 \beta_2} \right)^2 \beta_2} \right)^{1/2},$$

$$\text{Optimal mean quality: } \mu_2^* = \left(\frac{c}{\beta_2 + \left(\frac{\alpha_1 \beta_2}{\alpha_2 \beta_1} \right)^2 \beta_1} \right)^{1/2}.$$

Substituting the empirical values, $\alpha_1 = 0.229$, $\alpha_2 = 1.068$, $\beta_1 = 0.629$, $\beta_2 = 0.899$, and $c = 19.321$, we obtain,

$$\mu_1^* \cong 1.37$$

and

$$\mu_2^* \cong 4.49,$$

which are the empirical values of optimal mean quantity and quality for the health care sector under consideration.

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